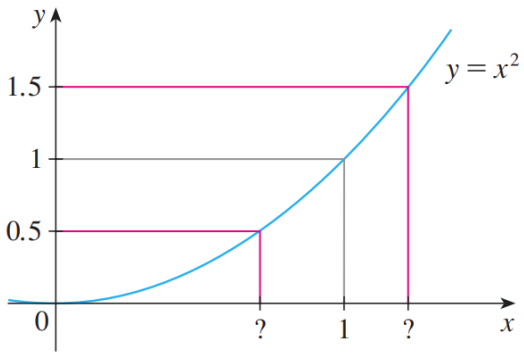
Exercise 4

Use the given graph of $f(x) = x^2$ to find a number δ such that

$$\text{if} \quad |x-1| < \delta \quad \text{then} \quad |x^2-1| < \frac{1}{2}$$

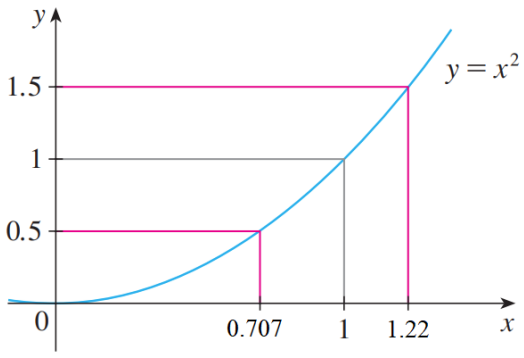


Solution

Start by finding the positive values of x that give $f(x) = 0.5$ and $f(x) = 1.5$.

$$x^2 = 0.5 \quad \rightarrow \quad x = \sqrt{0.5} = \frac{1}{\sqrt{2}} \approx 0.707$$

$$x^2 = 1.5 \quad \rightarrow \quad x = \sqrt{1.5} = \sqrt{\frac{3}{2}} \approx 1.22$$



As long as δ is less than $\sqrt{\frac{3}{2}} - 1 \approx 0.22$, the distance from 1 on the y-axis will be less than 0.5.